

Relativistic shock breakouts - a variety of gamma-ray flares: from low luminosity gamma-ray bursts to type Ia supernovae

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ABSTRACT

The light from shock breakout of stellar explosions carries a wealth of information on the progenitor and the explosion. The observed breakout signature strongly depends on the velocity of the shock at the time of breakout. The emission from Newtonian breakouts, typical in regular core-collapse supernovae (SNe), was explored extensively. However, a large variety of explosions result in mildly or ultra relativistic breakouts, where the observed signature is unknown. Here we calculate the luminosity and spectrum produced by relativistic breakouts. In order to do so we improve analytic description of relativistic radiation mediated shocks and follow the system from the breakout itself, through the planar phase and into the spherical phase. We limit our calculation to cases where the post breakout acceleration of the gas ends during the planar phase (i.e., the final gas Lorentz factor $\lesssim 30$). We find that spherical relativistic breakouts produce a flash of gamma-rays with energy, E_{bo} , temperature, T_{bo} , and duration, t_{bo}^{obs} , that provide the breakout radius ($\approx 5R_{\odot}(t_{bo}^{obs}/10 \text{ s})(T_{bo}/50 \text{ keV})^2$) and Lorentz factor ($\approx T_{bo}/50 \text{ keV}$) and that always satisfy $(t_{bo}^{obs}/20 \text{ s}) \sim (E_{bo}/10^{46} \text{ erg})^{1/2} (T_{bo}/50 \text{ keV})^{-2.68}$. The breakout flare is typically followed, on longer time scales, by X-rays that carry a comparable energy. We apply our model to a variety of explosions, including Ia and .Ia SNe, AIC, energetic SNe and gamma-ray bursts (GRBs). We find that all these events produce detectable gamma-ray signals, some of which may have already been seen. Some particular examples are: (i) Relativistic shock breakouts provide a natural explanation to the energy, temperature and time scales, as well as many other observed features, of *all* low luminosity GRBs. (ii) Nearby broad-line Ib/c (like SN 2002ap) may produce a detectable γ -ray signal. (iii) Galactic Ia SNe produce detectable γ -ray flares, if their progenitors are single degenerate. We conclude that relativistic shock breakout is a generic process for the production of gamma-ray flares, which opens a new window for the study and detection of a variety of stellar explosions.

1. Introduction

The first observable light from any stellar explosion is generated by the breakout of the shock that traverse the star. In most stellar objects the density drops sharply near the stellar edge. The shock that encounters this density drop accelerates, leaving most of its energy behind. Therefore, it breaks out at a velocity that is significantly higher than the typical shock velocity in the stellar interior, carrying only a small fraction of the total explosion energy. In typical core-collapse supernovae (SNe), where the total explosion energy is $\sim 10^{51}$ erg and the progenitor is not a compact object, the velocity of the shock at breakout is Newtonian. For example in a red supergiant progenitor the breakout velocity is in the range $5,000 - 10,000$ km/s, while in a Wolf-Rayet progenitor it is $\sim 30,000 - 100,000$ km/s. In cases where the explosion is more energetic or the progenitor is a compact object, the breakout can become mildly or even ultra relativistic.

Before its breakout, the shock is dominated by radiation. During Newtonian breakout all the photons that are contained in the shock transition layer are escaping the system and are seen as a short and bright flash. The properties of Newtonian shock breakout flashes were explored by many authors (e.g., Colgate 1974; Falk 1978; Klein & Chevalier 1978; Imshennik, Nadezhin & Utrobin 1981; Ensmann & Burrows 1992; Matzner & McKee 1999; Katz, Budnik & Waxman 2010; Nakar & Sari 2010; Katz, Sapir & Waxman 2011). These properties depend mostly on breakout radius and on the shock velocity at the layer that dominates the breakout emission, v_0 . Among these properties the observed typical photon frequency, denoted here as the radiation temperature T_0 , can be especially sensitive to the shock velocity. In relatively slow shocks, $v_0 < 15,000$ km/s, the radiation in the shock breakout layer is in thermal equilibrium and $T_0 \propto v_0^{1/2}$, producing typically a spectrum that peaks in the UV. However, as was shown by Weaver (1976), in faster radiation mediated shocks the radiation is away from thermal equilibrium in the immediate shock downstream, and as a result the temperature in the shock transition layer is much higher than the one obtained assuming thermal equilibrium. As a result the breakout in fast Newtonian shocks, $v_0 \sim 30,000 - 100,000$ km/s, peaks in X-rays (Katz, Budnik & Waxman 2010; Nakar & Sari 2010).

The emission following the breakout can be divided into two dynamically different regimes. At first, before the expanding gas doubles its radius, the dynamics is planar while at later time the evolution is spherical (Piro, Chang & Weinberg 2010; Nakar & Sari 2010; Katz, Sapir & Waxman 2011). The evolution of the temperature and luminosity during the planar phase was studied by Nakar & Sari (2010), where we have shown that if the breakout emission is out of thermal equilibrium at breakout, it remains out of equilibrium also during the planar phase and thermal equilibrium is achieved only during the spherical phase.

The evolution during the spherical homologous phase, was studied by many authors (e.g., Grassberg, Imshennik & Nadyozhin 1971; Chevalier 1976, 1992; Chevalier & Fransson 2008; Piro, Chang & Weinberg 2010; Rabinak & Waxman 2010; Nakar & Sari 2010), where all use the assumption (that in Nakar & Sari 2010 we prove to be adequate) of thermal equilibrium. All these studies considered only explosions where the shock breakout velocities are Newtonian and pairs are not significantly produced in the transition layer, i.e., $\beta_0 = v_0/c < 0.5$ where c is the speed of light.

A large range of cosmic explosions are expected to produce breakout velocities that are either mildly relativistic or ultra relativistic. A few examples are a range of white dwarf explosions such as Ia SN (Piro, Chang & Weinberg 2010), .Ia SN (Shen *et al.* 2010) and possibly accretion induced collapse (Dessart *et al.* 2006; Piro, Quataert & Metzger 2010). The energy involved in most of these explosions is $\lesssim 10^{51}$ erg, but the compact progenitor and relatively low ejecta mass results in relativistic breakouts. Another example is extremely energetic explosions of massive stars, with explosion energies in the range $10^{52} - 10^{53}$ erg. These were recently observed in growing numbers, e.g., SN2005ap and SN2007bi (Gal-Yam *et al.* 2009; Quimby *et al.* 2009, e.g.), and are believed to be the outcomes of various progenitor systems. Yet another example is massive star explosions that are associated with long gamma-ray bursts, where the breakout is ultra-relativistic. Relativistic breakouts may also take place in choked GRBs, where the relativistic jets fail to penetrate through the stellar envelope, and no regular long GRB is produced. As we show here, this mechanism is a promising source of all the observed low-luminosity GRBs.

The physical processes involved in producing the observed signature of a relativistic shock breakout is different from the Newtonian case in two fundamental ways. First, once the velocity of a radiation mediated shock becomes large enough, $\beta_s > 0.5$, and the temperature behind the shock exceeds 50 keV, pair production becomes the dominant source of leptons, which in turn are the main source of photons via free-free emission. The exponential sensitivity of the number of pairs to the temperature regulates the rest frame temperature in the shock immediate downstream to an almost constant value of 100 – 200 keV, independent of the shock Lorentz factor (Katz, Budnik & Waxman 2010; Budnik *et al.* 2010). The gas becomes loaded with pairs that dominate its optical depth, resulting in a temperature dependent opacity per unit of mass. This is in sharp contrast to Newtonian shocks, especially when $0.05 < \beta_s < 0.5$, where the temperature of the shocked fluid depends strongly on the shock velocity but the opacity per unit of mass is constant. The second major difference of relativistic explosions is the hydrodynamics both before and after the breakout. A Newtonian shock that propagates in a power-law decreasing density assumes the self similar solution of Sakurai (1960) before breakout, and the gas velocity is roughly constant after the breakout (it doubles by about a factor of two). Once the shock becomes relativistic it

follows the solution of Johnson & McKee (1971) (see also Tan, Matzner & McKee 2001 and Pan & Sari 2006) before the breakout and each fluid element continues to accelerate significantly following the breakout. These two differences, together with the usual relativistic effects such as relativistic beaming, govern the observed emission, which as we show here is very different than in the Newtonian case.

Yet another difference between Newtonian and relativistic radiation mediated shocks is the physical width of the shock. In radiation mediated shocks $\tau_s \sim c/v_s$, where τ_s is the total optical depth (including pairs and Klein-Nishina effects) seen by a photon going from the shock downstream towards the shock upstream. In Newtonian radiation mediated shocks, where no pairs are produced, the optical depth of a layer remains similar before it is shocked and once it becomes the shock transition layer. Therefore, the shock breaks out at the layer in which $\tau = \tau_s$, where τ is the Thompson optical depth to the stellar edge *before* the shock crossing (i.e., without pairs). However, in relativistic shocks, where pairs are produced and Klein-Nishina effects may become important, the optical depth of a layer is changed significantly when it is swept up by the shock and the relation between τ_s and τ is not trivial anymore. In order to find this relation one needs to know the structure of the shock transition layer. The structure of relativistic radiation mediated shocks in different regimes was solved numerically by Budnik *et al.* (2010) and Levinson & Bromberg (2008). The solution of Budnik *et al.* (2010), where a significant number of photons is generated within the shock, is the relevant one to the type of shocks we consider here (Bromberg, Mikolitzky & Levinson 2011). Budnik *et al.* (2010) also provide an approximate analytic description of the shock structure as function of the optical depth. We derive a more accurate analytic description of the structure of the shock transition layer and use it to find the value of τ at the point that the shock breaks out. We find that while production of pairs results in a shock that breaks out of the star at $\tau \ll 1$, the observed emission is always dominated by the layer where $\tau \approx 1$. We therefore use the subscript ‘₀’ and the terminology ‘breakout shell’ to denote the $\tau = 1$ layer, which is not the actual layer where the breakout takes place, but is the layer that dominates the observed breakout emission.

In this paper we calculate the evolution of the observed luminosity and temperature from explosions in which the shock becomes mildly or ultra relativistic, i.e., $\gamma_0\beta_0 > 0.5$. We follow the light curve as long as the observed radiation is generated by gas that is moving at relativistic velocities. At later times the radiation is determined by Newtonian gas, which light curve was discussed in Nakar & Sari (2010). Our solution is limited to cases where the breakout shell ends its post shock acceleration before it doubles its radius. This limit is translated to a final (post-acceleration) Lorentz factor of the breakout shell $\lesssim 30$. We also assume that the opacity of the progenitor wind is negligible and do not affect the breakout emission. Our calculations are applicable to a wide range of energetic and/or compact

explosions including Ia and .Ia SNe, energetic core collapse and pair instability SNe and possibly part of the phase space of choked GRB jets.

We find that typically, relativistic breakouts produce flashes of gamma-rays that without further information (e.g., the burst redshift) can be mistaken as cosmological gamma-ray bursts. These results provide a closure to Colgate (1968), who was the first to predict that shock breakouts produce flares of gamma-rays, even before the detection of gamma-ray bursters was announced (Klebesadel, Strong & Olson 1973). However, Colgate (1968) assumed that the gas behind the shock is in thermal equilibrium, and the observed γ -rays are due to blue shift of ultra-relativistic gas, brought to high Lorentz factors by shock that accelerates indefinitely as it approaches the stellar surface. It ignores deviation from thermal equilibrium and the fact that acceleration stops once the shock width is comparable to the distance to the stellar surface. As it turned out, the deviation from thermal equilibrium increases the photon’s rest frame temperature and compensates for the limited Lorentz factor of the gas, resulting in a burst of γ -rays. Since Colgate’s prediction, breakouts were suggested to produce bright gamma-ray emission by several authors in the context of low-luminosity GRBs (Kulkarni *et al.* 1998; Tan, Matzner & McKee 2001; Campana *et al.* 2006; Waxman, Mészáros & Campana 2007; Wang *et al.* 2007). All these calculations assume thermal equilibrium, underestimating the true radiation temperature. Katz, Budnik & Waxman (2010) realized that the deviation from equilibrium, discussed by Weaver (1976), will lead to γ -ray breakouts. They did not follow, however, the post shock dynamics and opacity evolution to find the properties of the observed emission. Here we provide, for the first time, a quantitative description of the predicted light curve from relativistic breakouts. We show that these light curves explain very well many of the observed features of low-luminosity GRBs, and also, that a variety of other stellar explosions are most likely producing a bright gamma-ray breakout emission.

In §2 we discuss the pre and post breakout dynamics. The dynamics is dominated by mass, energy and momentum conservation and is therefore largely independent of the photon number and spectrum, as long as the gas is optically thick. In §3 we calculate the temperature and opacity evolution of the expanding gas. §4 follows the observed light curve from the breakout to the spherical phase. Our results are applied in §5 to find the observed signature of known and hypothesized explosions. An analytic description of the structure of the transition layer in relativistic radiation mediated shocks is presented in Appendix A.

2. Dynamics

2.1. Pre-breakout dynamics

Consider a relativistic blast wave that propagates near a stellar edge pre-explosion mass density gradient of the form $\rho \propto z^n$, where $z = (R_* - r)$, R_* is the stellar radius and r is the distance from the star center. In compact stars, where shocks are more likely to become relativistic, the envelope is radiative and typically $n \approx 3$, which is the value that we use throughout the paper. The shock acceleration stops when the width of the shock is comparable to z and the shock breaks out. As we show in appendix A, this takes place in the shell where the pre-explosion optical depth to the stellar edge is $\tau \sim 0.01\gamma_s$, where γ_s is the shock Lorentz factor. Namely, breakout takes place at $z \sim 0.01\gamma_s/(\rho\kappa_T)$, where $\kappa_T \approx 0.2 \text{ cm}^2/\text{g}$ is the Thompson cross section per unit of mass¹. This sets the maximal Lorentz factor of the shock and we refer to that shell as the *outermost shell*. Unlike the Newtonian case, the observed emission from relativistic breakout is not dominated by the outermost shell. Instead it is dominated by the shell where the pre-explosion optical depth is² $\tau \sim 1$ (i.e., $z \sim 1/(\rho\kappa_T)$), which we therefore refer to as *the breakout shell* and denote its properties by the subscript ‘₀’. In case of properties that evolve with time, this subscript refers to their value in the breakout shell right after it is crossed by the shock. Given the above assumptions the system is completely defined by the following parameters of the breakout shell:

- γ_0 : the Lorentz factor of the shock when it crosses the breakout shell.
- m_0 : the mass of the breakout shell.
- d_0 : the post-shock width (i.e., after compression) of the breakout shell in the lab frame.
- R_* : the radius of the star.
- n : the power law index describing the pre-explosion density profile, here we use $n = 3$.

¹Throughout the paper we use this value, which is appropriate for hydrogen free ionized gas. The dependence on the value of κ_T is weak and all our results are applicable also to hydrogen rich ionized gas. Note that through most of the phases discussed here the temperatures in the system are larger than 10 keV, where the gas is fully ionized regardless of its composition, implying that the results are insensitive to metallicity.

²This is the criterion for the dominating shell as long as $\gamma_0 \lesssim 100$. As we discuss below this criterion is always satisfied in the regime at which our analysis is applicable

Other characteristics of the breakout shell can be calculated from the above. For example, the initial thermal energy of the breakout shell is $E_0 \approx \gamma_0^2 m_0 c^2$.

The pre-breakout and post-breakout dynamics of a planar relativistic blast-wave in a decreasing density profile was investigated by Johnson & McKee (1971), Tan, Matzner & McKee (2001) and Pan & Sari (2006). If the density follows a power-law then the evolution is self similar both before and after the breakout. Before breakout a relativistic shock accelerates as $\gamma_i \propto \rho^{-\mu}$ where $\mu = (\sqrt{3} - \frac{3}{2}) \approx 0.23$ and the subscript ‘ i ’ indicates initial values (note that ρ is the pre-shock density). This propagation profile sets the entire pre-explosion hydrodynamic profile. Conceptually, it is useful to treat the whole expanding envelope as a series of successive shells. The properties of any shell deeper than the outermost shell, can be characterized by its mass, m :

- $\gamma_i = \gamma_0(m/m_0)^{\frac{-\mu n}{n+1}} \propto m^{-0.17}$ is the initial Lorentz factor.
- $d_i \approx z/\gamma_i^2 \propto m^{\frac{1+2n\mu}{(n+1)}} \approx m^{0.6}$ is the width of a shocked shell at shock breakout, in the lab frame.
- $d'_i = d_i \gamma_i \propto m^{\frac{1+n\mu}{(n+1)}} \approx m^{0.42}$ is the width of a shocked shell at shock breakout, in the shell rest frame.
- $E_i \approx \gamma_i^2 m c^2 \propto m^{\frac{1+n(1-2\mu)}{n+1}} \approx m^{0.65}$ is the internal energy of a shell at shock breakout, in the lab frame.
- $t_i \approx \frac{d_i}{c} \gamma_i^2 \propto m^{\frac{1}{n+1}} \propto m^{0.25}$ is the lab frame time for the shell expansion, which is also the time between the shock crossing and the breakout

2.2. Post-breakout dynamics

Following breakout all the shells expand and accelerate. The acceleration is faster than that of a freely expanding relativistic shell as outer shells gain energy from inner ones. In this subsection we follow the acceleration of shells with mass $m \gtrsim m_0$. These shells remain opaque during the whole planar phase, also after their pairs annihilate. The final Lorentz factor of less massive shells depends on their evolving opacity and is discussed shortly in the next section.

The acceleration during the planar phase, i.e., before a shell doubles its radius, follows (Johnson & McKee 1971; Pan & Sari 2006):

$$\gamma = \gamma_i \left(\frac{t}{t_i} \right)^{\frac{\sqrt{3}-1}{2}} \propto m^{-0.27} t^{0.37}, \quad (1)$$

where t is the lab frame time (not to be confused with observer time) measured since breakout. If the shell is optically thick at the end of acceleration and acceleration ends during the planar phase then the final Lorentz factor is

$$\gamma_f = \gamma_i^{1+\sqrt{3}} \propto m^{-0.48}. \quad (2)$$

This relation is general as it does not depend on the exact density profile, as long as there is a large energy reservoir behind the accelerating shell. The acceleration ends at

$$t_f = t_i \gamma_i^{3+\sqrt{3}} \propto m^{-0.57}. \quad (3)$$

Thus, more massive shells end their acceleration at earlier times and lower Lorentz factors.

In this paper we restrict our treatment to cases where the breakout shell ends its acceleration during the planar phase, i.e., $t_{f,0} < t_s$ where

$$t_s = \frac{R_*}{c} \quad (4)$$

is the lab frame time of transition between the planar and spherical phases. Since pre-shocked optical depth of the breakout shell is of order unity (i.e., $\kappa_T \rho_0 z_0 \approx 1$) :

$$\frac{t_0}{t_s} = \frac{z_0}{R_*} \approx \left(\frac{R_*^2}{\kappa_T M_*} \right)^{\frac{1}{n+1}} = 3 \cdot 10^{-3} M_5^{-0.25} R_5^{0.5} \quad (5)$$

where M_x and R_x are the progenitor star mass and radius in units of x solar masses and radii respectively. Thus, there is a critical Lorentz factor, $\gamma_{0,s}$, for which $t_{f,0} = t_s$:

$$\gamma_{0,s} = 3.5 M_5^{0.05} R_5^{-0.1}. \quad (6)$$

The corresponding critical final Lorentz factor of the breakout shell is:

$$\gamma_{f,s} = \gamma_{0,s}^{\sqrt{3}+1} = 30 M_5^{0.14} R_5^{-0.27} \quad (7)$$

If $\gamma_0 \leq \gamma_{0,s}$ then acceleration ends in the planar phase and the corresponding final Lorentz factor is given by equation 2. Otherwise it ends during the spherical phase and equation 2 is not valid anymore, a case that we treat elsewhere.

Acceleration continues beyond the breakout shell, up to the outermost shell, which satisfies $z \sim 0.01 \gamma_i / (\rho \kappa_T)$. This shell achieves the maximal initial Lorentz factor:

$$\gamma_{max,i} \approx 2 \gamma_0^{0.85} \quad (8)$$

Note that our calculation is limited to $\gamma_0 < \gamma_{0,s} \ll 100$, where $\gamma_0 < \gamma_{max,i}$ and the entire above description is consistent.

3. Temperature and opacity

The rest frame temperature behind a radiation mediated shock with $\beta_s > 0.5$ is roughly constant³ (Katz, Budnik & Waxman 2010; Budnik *et al.* 2010):

$$T'_i \sim 200 \text{ keV} \quad (9)$$

The reason is that at these velocities the radiation behind the shock is out of thermal equilibrium and is set by the ability to generate photons. The main photon source in the shock is free-free emission (Weaver 1976). Once the temperature behind the shock rises above ≈ 50 keV, pairs are generated and their number becomes dominant over the number of electrons that are advected from the upstream. These pairs significantly increase the rate of photon generation. The strong dependence of the number of pairs on the temperature in that range serves as a "thermostat" that sets the temperature in the immediate shock downstream to be nearly constant, regardless of the shock Lorentz factor. Although the radiation is out of thermal equilibrium the photons and the pair loaded gas in the immediate downstream are in Compton-pair equilibrium and there is a single gas-radiation temperature (Budnik *et al.* 2010). This property of relativistic radiation mediated shocks is dominating the emission at the shock breakout, during the planar phase and the relativistic part of the spherical phase.

The gas behind the shock is pair loaded and therefore its optical depth is very high, preventing any significant leakage of photons, even if the pre-shocked (unloaded) gas optical depth is below unity (the pairs set a diffusion time that is much larger than the dynamical time). On the other hand, the rest frame temperature in the expanding gas drops, and with it the number of pairs and the photon generation rate. Therefore, the number of photons in the opaque expanding shells is roughly constant. The mildly relativistic temperature and the large optical depth ensure that photons and pairs interact many times (i.e., sharing energy, annihilation and creation) over the dynamical time, thereby keeping the pairs-photon gas in Compton pair equilibrium.

The radiation of the breakout shell is confined to the gas during its expansion until the rest frame temperature drops enough so the pair loading becomes negligible. During the expansion of a shell its rest frame temperature falls as $T' \propto V'^{-\beta}$, where V' is the shell volume (in its rest frame) and $1+\beta$ is an effective adiabatic index. The value of β depends on temperature and it can drop slightly (by up to 30%) below $1/3$ because of pairs production and annihilation. Here we neglect this small deviation from the typical relativistic equation

³This is true as long as the blackbody temperature that corresponds to the post shock energy density is lower than T'_i

of state and approximate $\beta = 1/3$. During the planar phase the volume of a fluid element grows as $V' \propto t/\gamma$. Therefore during acceleration $V' \propto t^{(3-\sqrt{3})/2}$ while after the acceleration ends $V' \propto t$. During the spherical phase $V' \propto t^3$ implying:

$$T' = T_i \begin{cases} \left(\frac{t}{t_i}\right)^{-\frac{3-\sqrt{3}}{6}} & t < t_f \\ \gamma_i^{-1} \left(\frac{t}{t_f}\right)^{-1/3} & t_f < t < t_s \\ \gamma_i^{-1} \left(\frac{t_s}{t_f}\right)^{-1/3} \left(\frac{t}{t_s}\right)^{-1} & t_s < t \end{cases} \quad (10)$$

The optical depth of the breakout shell drops to unity once the pairs density, n_{\pm} , is lower than the proton density, n_p (and their accompanied electrons), i.e., $n_{\pm}/n_p < 1$. The initial value of this ratio is (e.g., Svensson 1984; Budnik *et al.* 2010):

$$\frac{n_{\pm,0}}{n_{p,0}} \approx \frac{\gamma_0 m_p c^2}{3kT'_i} \approx 10^3 \gamma_0 \quad (11)$$

Since pairs are in annihilation-creation equilibrium their density drops exponentially with T' at this temperature range and the shell opacity is dominated by the electrons that were advected from the upstream once its temperature is:

$$T'_{th} \approx 50 \text{ keV}, \quad (12)$$

where the dependence on γ_0 is very weak. Around this temperature, the breakout shell, and any less massive shell, becomes transparent and its radiation escapes. We define $t_{th,0}$ as the time that the breakout shell becomes optically thin, i.e., its $T' = T'_{th}$.

There are three critical times in the system, in which the dynamics and radiation change their behavior. These are the acceleration and transparency times of the breakout shell, $t_{f,0}$ and $t_{th,0}$ respectively, and the planar to spherical transition time, t_s . The observed signal depends on the relative order of these three time scales. Here we restrict our analysis to the case that acceleration ends during the planar phase, i.e., $t_{f,0} < t_s$. Under this assumption, equation 10 implies:

$$t_{th,0} = \begin{cases} t_0 \left(\frac{T'_0}{T'_{th}}\right)^{3+\sqrt{3}} & \frac{T'_0}{T'_{th}} < \gamma_0 & (t_{th,0} < t_{f,0}) \\ t_0 \left(\frac{T'_0}{T'_{th}}\right)^3 \gamma_0^{\sqrt{3}} & \gamma_0 < \frac{T'_0}{T'_{th}}, \left(\frac{t_s T'^3_{th}}{t_0 T'^3_0}\right)^{\frac{\sqrt{3}}{3}} & (t_{f,0} < t_{th,0} < t_s) \\ t_s \frac{T'_0}{T'_{th}} \gamma_0^{\frac{\sqrt{3}}{3}} \left(\frac{t_0}{t_s}\right)^{\frac{1}{3}} & \left(\frac{t_s T'^3_{th}}{t_0 T'^3_0}\right)^{\frac{\sqrt{3}}{3}} < \gamma_0 < \frac{T'_0}{T'_{th}} & (t_s < t_{th,0}) \end{cases} \quad (13)$$

Now, $T'_0/T'_{th} \sim 4$ and we consider only $\gamma_0 < \gamma_{0,s}$, which for any non-degenerate progenitor implies $\gamma_0 < \gamma_{0,s} \lesssim 4$. Therefore, the breakout shell becomes transparent only after acceleration ends, i.e., $t_{f,0} < t_{th,0}$. On the other hand, the opacity drops quickly during the spherical

phase, so equation 13 implies that the breakout shell becomes transparent during the planar phase, or at the beginning of the spherical phase, namely $t_{th,0} \lesssim t_s$. Therefore for a large range of realistic explosion parameters $t_{f,0} < t_{th,0} < t_s$, which is the case that we present in detail below. Finally, note that shells with $\gamma_i > \gamma_0$ can be accelerated only as long as they are optically thick, namely their $T' > T'_{th}$. Therefore, only shells with $\gamma_i \lesssim \min\{4, \gamma_{0,s}\}$ satisfy the relation $\gamma_f = \gamma_i^{1+\sqrt{3}}$, shells with higher γ_i end their acceleration before this relation is satisfied.

4. Light curve

4.1. Breakout emission

The radiation in the expanding gas is trapped by pairs at the time of breakout. In the previous section we have shown that for most progenitors in the regime that we consider, the breakout shell becomes transparent after it ended acceleration and before, or soon after, the transition to the spherical phase, i.e., $t_{f,0} < t_{th,0} \lesssim t_s$. Therefore we derive below the observed light curve in that case. Note that the breakout shell is not the only shell that becomes transparent during the planar phase. All the lighter shells ($m < m_0$) out to the the outermost shell, become transparent during the planar phase as well. Shells that are deeper (and more massive and slower) than the breakout shell remain opaque during the planar phase and become transparent only during the spherical phase.

Since slower moving shells carry significantly more energy, $E_i \propto \gamma_i^{-3.75}$, the breakout emission is dominated by the energy confined to the breakout shell at $t_{th,0}$, when its local frame temperature is T'_{th} :

$$E_{bo} = E_0 \frac{\gamma_{f,0}}{\gamma_0} \frac{T'_{th}}{T'_0} \approx \frac{1}{4} m_0 c^2 \gamma_0 \gamma_f \approx 2 \cdot 10^{45} R_5^2 \gamma_f^{\frac{1+\sqrt{3}}{2}}, \quad (14)$$

where we use $\tau_0 \approx \kappa_T m_0 / (4\pi R_*^2) = 1$ to find

$$m_0 \approx 4 \cdot 10^{-9} M_\odot R_5^2. \quad (15)$$

The observed temperature of this emission is

$$T_{bo} = T'_{th} \gamma_{f,0} \sim 50 \gamma_f \text{ keV}, \quad (16)$$

and it is smeared by light travel time effects over a duration that is comparable to t_s^{obs} :

$$t_{bo}^{obs} \approx t_s^{obs} \approx \frac{R_*}{c \gamma_{f,0}^2} \approx 10 \frac{R_5}{\gamma_{f,0}^2} \text{ s}, \quad (17)$$

where t_s^{obs} is the time of transition from the planar to the spherical phase as seen by the observer. Clearly, equations 15-17 are general and are applicable to any gas density profile. In fact also equation 14 is applicable to any sharply decreasing density gradient, even if it is not a power-law. The reason is that m_0 does not depend on this profile and so does the relation between γ_0 and $\gamma_{f,0}$ (equation 2), as long as acceleration ends during the planar phase and there is a large energy reservoir behind the breakout shell. Together, equations 14-17 provide three generic observables that depend on only two parameters $\gamma_{f,0}$ and R_* . Thus, the energy, temperature and time scale of the breakout flare are enough to determine R_* and $\gamma_{f,0}$, and they also must satisfy

$$t_{bo}^{obs} \sim 20 \text{ s} \left(\frac{E_{bo}}{10^{46} \text{ erg}} \right)^{1/2} \left(\frac{T_{bo}}{50 \text{ keV}} \right)^{-\frac{9+\sqrt{3}}{4}}, \quad (18)$$

if the source of the flare is a quasi-spherical⁴ relativistic shock breakout. This relativistic breakout relation can be used to test the origin of observed gamma-ray flares.

During $t < t_{bo}^{obs}$ the observer receives also photons from shells that are outer of the breakout shell, which have higher observed temperature than T_{bo} . These photons generate a high energy power-law at frequencies $T_{bo} < \nu$. Shells with $\gamma_i < T'_i/T'_{th} \approx 4$ obtain their terminal Lorentz factor before they become optically thin, implying that $\gamma_f = \gamma_i^{\sqrt{3}+1}$ and $E \propto E_i \gamma_f / \gamma_i \propto \gamma_i^{-2.01}$ while $T = T'_{th} \gamma_f \propto \gamma_i^{\sqrt{3}+1}$. Therefore the integrated spectrum of the breakout flare exhibit a high energy power-law

$$\nu F_\nu(\nu > T_{bo}) \propto \nu^{-0.74} \quad (19)$$

up to $\nu = \min\{2, 0.25\gamma_0^{2.05}\} \text{ MeV}$, which can be an order of magnitude larger than T_{bo} . During $t_{obs} < t_{bo}^{obs}(= t_s^{obs})$ the observer receives also photons from the breakout shell itself after it becomes transparent. During this phase adiabatic cooling dictates (e.g., Nakar & Sari 2010), $T \propto t_{obs}^{-1/3}$ and $L \propto t_{obs}^{-4/3}$, implying

$$\nu F_\nu(\nu < T_{bo}) \propto \nu \quad (20)$$

The range of the low energy power-law is rather limited $T_{bo} > \nu > T_{bo}(ct_{th,0}/R_*)^{1/3} = 0.5T_{bo}\gamma_0^{1/\sqrt{3}}M_5^{-1/12}R_5^{1/6}$. Note that equation 19 and the spectral range where the high and low energy power-laws are observed, do depend on the specific density profile and are given for $n = 3$.

⁴ E_{bo} , T_{bo} , and t_{bo}^{obs} depend only on emission from a region within an angle of $\sim 1/\gamma_{f,0}$ with respect to the line of sight

At t_s^{obs} the observed luminosity is still dominated by light emitted when the breakout shell become transparent, $\sim t_{th,0}$. This emission fades quickly until the emission from the spherical phase becomes dominant. The fading light curve can be easily derive in case that $\gamma_{f,0} \gg 1$, since then the observed luminosity is dominated by emission of the breakout shell at large angles ($> 1/\gamma_{f,0}$). The luminosity decays then as $L \propto t_{obs}^{-2}$ and the temperature as $T \propto t_{obs}^{-1}$ (Kumar & Panaitescu 2000; Nakar & Piran 2003), until the emission from the spherical phase becomes dominant.

4.2. Spherical phase

During the spherical phase the radius of the expanding sphere is not constant anymore. Instead, the radius of the shells is $r \propto vt$ and the luminosity is dominated by photons that are leaking from the shell that satisfies $\tau \sim c/v$. We refer to this shell as the *luminosity shell* and denote its properties with the superscript ‘ $\hat{}$ ’ (see Nakar & Sari 2010 for a detailed discussion), such that, e.g., the luminosity shell satisfies, by definition, $\hat{\tau} = c/\hat{v}$. The evolution during the spherical phase depends on whether the luminosity shell is relativistic or not and on the initial temperature of the luminosity shell. As long as $\hat{\gamma}_f \gg 1$ the initial temperature of the luminosity shell is ~ 200 keV and its dynamics is well approximated by the relativistic self-similar solution. Shells with $0.5 < \gamma_i \beta_i < 1$ also have an initial temperature of ~ 200 keV but their dynamics is better described by Newtonian approximation. Namely, shock propagation according to the self similar solution of Sakurai (1960) and no significant acceleration after shock breakout. The transition time between the relativistic and Newtonian phases is calculated below and it takes place around:

$$t_{NW}^{obs} = \frac{R_*}{c} \gamma_{f,0}^{1.05} = t_s^{obs} \gamma_{f,0}^{3.05} \quad (21)$$

During the Newtonian phase $t_{obs} \propto \hat{v}^{-4}$. The end of the phase in which the initial temperature of the luminosity shell is constant takes place when $\hat{v} \approx 0.5c$, which is at $t_{obs} \approx 10t_{NW}^{obs}$. Here we present luminosity and temperature evolution during the spherical phase up to that point ($\hat{v} \approx 0.5c$). At later times the temperature drops quickly until thermal equilibrium is obtained. Therefore there is a sharp break in the temperature evolution at $10t_{NW}$. The light curve evolution at $t > 10t_{NW}$ is covered by Nakar & Sari (2010).

4.2.1. Relativistic phase ($t_s^{obs} < t_{obs} < t_{NW}^{obs}$)

The relativistic velocity of the shells dictates $\hat{\tau} \sim 1$ and since $\tau \propto m/r^2$ the mass of the luminosity shell is $\hat{m} \propto t^2$. The photons from the luminosity shell arrive to the observer

at $t_{obs} \approx t/\hat{\gamma}_f^2$ and their arrival is distributed over a comparable duration. Therefore the luminosity shell satisfies $\hat{m} \propto t_{obs}^{0.69}$, $\hat{\gamma}_i \propto t_{obs}^{-0.12}$, $\hat{\gamma}_f \propto t_{obs}^{-0.33}$ and $\hat{E}'_i \propto \hat{m}\hat{\gamma}_i \propto t_{obs}^{0.57}$, while $\hat{r} \propto t \propto t_{obs}\hat{\gamma}_f^2 \propto t_{obs}^{0.34}$ and equation 10 dictates $\hat{T}' \propto t_{obs}^{-0.36}$. The observed luminosity and temperature are:

$$\hat{L} = \frac{\hat{E}}{t_{obs}} = \frac{\hat{\gamma}_f \hat{E}'_i (\hat{T}'/T'_i)}{t_{obs}} = \frac{E_0 c}{R_*} \left(\frac{t_0}{t_s}\right)^{1/3} \gamma_0^{\frac{4\sqrt{3}+9}{3}} \left(\frac{t_{obs}}{t_s^{obs}}\right)^{-1.12} \quad (22)$$

$$\hat{T} \approx 200 \text{ keV} \left(\frac{t_0}{t_s}\right)^{1/3} \gamma_0^{\frac{\sqrt{3}}{3}} \left(\frac{t_{obs}}{t_s^{obs}}\right)^{-0.68} \quad (23)$$

The relativistic phase ends once $\hat{\gamma}_i \approx 1$ at a time given in equation 30.

4.2.2. Newtonian phase with constant initial temperature ($t_{NW}^{obs} < t_{obs} < 10 t_{NW}^{obs}$)

The luminosity during this phase is independent of the temperature since pairs are long gone. It is therefore similar to the luminosity evolution of the spherical phase following a Newtonian breakout, where:

$$\hat{L} \propto t_{obs}^{-0.35} \quad (24)$$

Therefore a significant flattening of the luminosity evolution is expected at t_{NW} . From this point the luminosity decay is steady until recombination and/or radioactivity start playing a role.

The initial volume of the luminosity shell is proportional to its initial width (all shells starts at R_*), $\hat{d}_i \propto t^{0.44}$. The observed temperature evolves as:

$$\hat{T} \approx T'_i \left(\frac{\hat{v}^3 t^3}{\hat{d}_i R_*^2}\right)^{-1/3} \propto t_{obs}^{-0.6} \quad (25)$$

This slope is not very different than that of the relativistic phase. Therefore there is no significant feature in the temperature evolution at t_{NW} . Only at $t_{obs} \sim 10 t_{NW}$ the temperature starts dropping rapidly until the luminosity shell gains thermal equilibrium.

5. Predicted signal of various explosions

We use the model developed in the previous section to calculate the signal expected from known and hypothesized explosions. There is a large number of explosions, such as

various SNe types, in which the bulk of the mass is moving in Newtonian velocities but the shock accelerates a minute amount of mass to a relativistic breakout. Such explosions are often observed mostly in the optical, where the kinetic energy, E , that is carried by the bulk of the ejected mass, M_{ej} , can be measured. Therefore we first provide the characteristic properties of the breakout and following emission of such explosions as a function of E , M_{ej} and R_* . Then we use these results to discuss the predicted signals from various explosions.

Given E , M_{ej} and R_* the initial Lorentz factor of the breakout shell is determined by following the shock acceleration from the Newtonian phase ($\gamma_s \beta_s \propto \rho^{-0.19}$), to the relativistic phase ($\gamma_s \beta_s \propto \rho^{-0.23}$):

$$\gamma_0 \beta_0 \approx 2.6 E_{53}^{0.62} M_{ej,5}^{-0.45} R_5^{-0.35}. \quad (26)$$

where $E_{53} = E/(10^{52} \text{ erg})$. The numerical coefficient is taken to fit the numerical results of Tan, Matzner & McKee (2001), which simulate the transition of the shock from Newtonian to relativistic. Equation 26 provides a good approximation to the breakout Lorentz factor for $\beta_0 > 0.5$. The initial energy of the breakout shell is

$$E_0 \approx 5 \cdot 10^{46} \text{ erg } E_{53}^{1.25} M_{ej,5}^{-0.9} R_5^{1.3}, \quad (27)$$

and its final Lorentz factor is

$$\gamma_{f,0} \approx 14 E_{53}^{1.7} M_{ej,5}^{-1.2} R_5^{-0.95} \quad (28)$$

where we use the approximation⁵ $\gamma_0 \approx \gamma_0 \beta_0$. Figure 1 shows a color map of the value of $\gamma_{f,0}$ as a function of ejected mass and stellar radius for different values of explosion energies, where only the range where our results are valid, $0.5 < \gamma_0 \beta_0 < \gamma_{0,s}$, is colored. The figure also includes the typical phase space location of various explosions in that range. It is evident that breakouts of white dwarf explosions, extremely energetic SNe and possibly low luminosity GRBs are all within the relevant velocity range.

⁵This approximation is good for $\gamma_0 \beta_0 > 2$. For lower values of $\gamma_0 \beta_0$ it is better to calculate $\gamma_{f,0} = \gamma_0^{1+\sqrt{3}} = (\gamma_0^2 \beta_0^2 + 1)^{\frac{1+\sqrt{3}}{2}}$ and plug it into equation 29

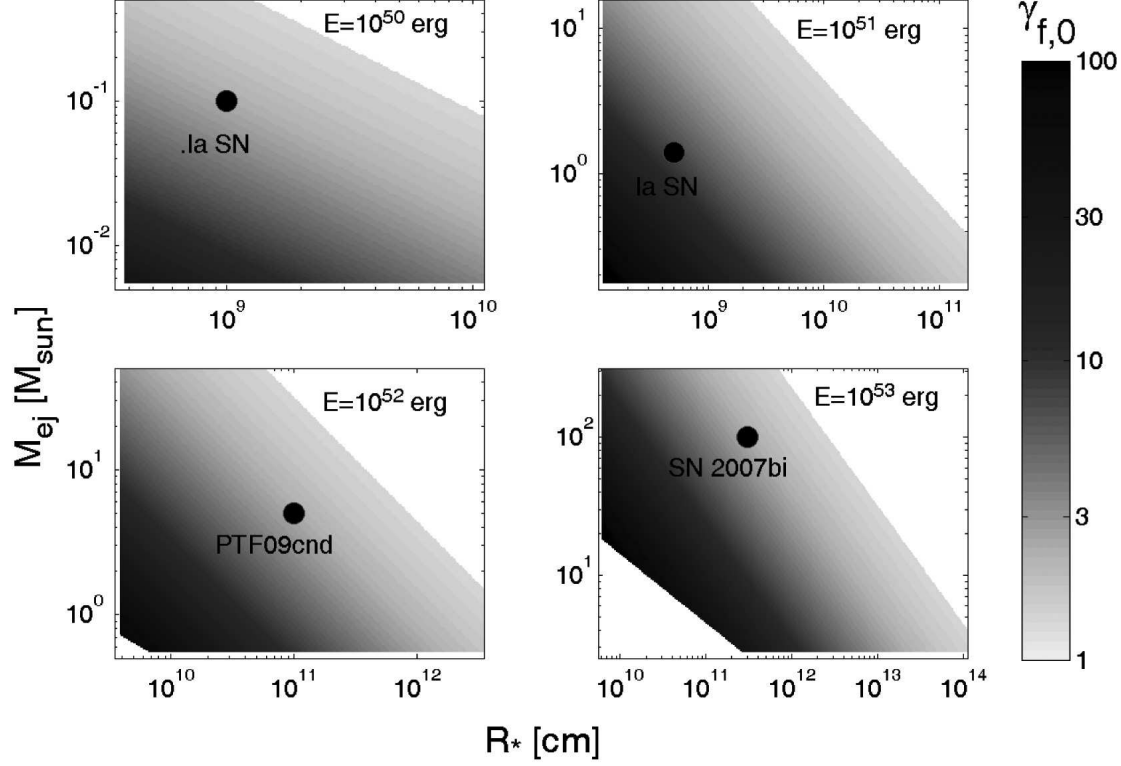


Fig. 1.— A color map of the value of $\gamma_{f,0}$ as function of ejected mass and stellar radius for different values of explosion energies. Only cases with $0.5 < \gamma_0 \beta_0 < \gamma_{0,s}$ are colored. The approximate phase space location of various explosions where the breakout velocity falls in this range are marked as well.

Plugging equation 28 into equations 14-17 we find the observed breakout emission:

$$\begin{aligned}
 E_{bo} &\approx \frac{E_0 \gamma_{f,0}^{\frac{3-\sqrt{3}}{2}}}{4} \approx 6 \cdot 10^{46} \text{ erg } E_{53}^{2.3} M_{ej,5}^{-1.65} R_5^{0.7} \\
 T_{bo} &\approx 50 \text{ keV } \gamma_{f,0} \approx 700 \text{ keV } E_{53}^{1.7} M_{ej,5}^{-1.2} R_5^{-0.95} \\
 t_{bo}^{obs} &\approx t_s^{obs} = \frac{R_*}{c \gamma_{f,0}^2} \approx 0.06 \text{ s } E_{53}^{-3.4} M_{ej,5}^{2.5} R_5^{2.9} \\
 L_{bo} &\approx \frac{E_{bo}}{t_{bo}^{obs}} \approx 4 \cdot 10^{47} \text{ erg/s } E_{53}^{5.1} M_{ej,5}^{-3.65} R_5^{-1.85}
 \end{aligned} \tag{29}$$

Note that this equation is applicable only to a power-law density profile ($n=3$) (unlike equations 14-17 which are general), due to the dependence of $\gamma_{f,0}$ on the density profile in case that E , M_{ej} and R_* are the physical parameters.

The breakout emission dominates during the entire planar phase. At the end of the planar phase (t_s^{obs}) the luminosity and temperature drops quickly ($L \propto t_{obs}^{-2}$, $T \propto t_{obs}^{-1}$ if $\gamma_{f,0} \gg 1$; see discussion in previous section) to join the spherical emission, which is dominated by relativistic ejecta until

$$t_{NW}^{obs} \approx 200 \text{ s } E_{53}^{1.8} M_{ej,5}^{-1.3}, \quad (30)$$

where we neglect a very weak dependence on R_* . The luminosity and temperature at that time are

$$L(t_{NW}^{obs}) \approx 3 \cdot 10^{44} \text{ erg } E_{53}^{0.9} M_{ej,5}^{-0.65} R_5^{0.65} \quad (31)$$

$$T(t_{NW}^{obs}) \approx 0.2 \text{ keV } E_{53}^{-3.15} M_{ej,5}^{2.3} R_5^{1.9}$$

where we ignore very weak dependence⁶ on M_* and use the approximation $\gamma_0 \approx \gamma_0 \beta_0$. At $t_s^{obs} < t_{obs} < t_{NW}^{obs}$ the luminosity shell is relativistic and $L \propto t_{obs}^{-1.12}$ while $T \propto t_{obs}^{-0.68}$. At $t_{NW}^{obs} < t_{obs} < 10t_{NW}^{obs}$ the luminosity shell is Newtonian and $L \propto t_{obs}^{-0.35}$ while $T \propto t_{obs}^{-0.6}$. The energy emitted per logarithmic scale in time, Lt_{obs} , is rising during that last phase ($\propto t_{obs}^{0.65}$), and most of the energy that is emitted in X-ray (for typical parameters) is emitted around $t_{obs} \sim 10t_{NW}^{obs}$. This energy may be comparable to, or even larger than, the breakout energy:

$$Lt_{obs}(10t_{NW}^{obs}) \approx 3 \cdot 10^{47} \text{ erg } E_{53}^{2.7} M_{ej,5}^{-1.9} R_5^{0.65} \quad (32)$$

At later time the luminosity evolution remains unchanged (until recombination or radioactive decay become dominant), while the temperature drops rapidly towards thermal equilibrium (Nakar & Sari 2010).

5.1. White dwarf explosions - Ia SN, .Ia SN and AIC

There are several mechanisms that can result in a shock breakout from a white dwarf. The most famous is type Ia SNe, where the whole star explodes. Other theoretically predicted scenarios are .Ia SNe (Shen *et al.* 2010), where helium accreted in an AM CVn is detonated,

⁶In some explosions $M_{ej} = M_*$ while in others $M_{ej} < M_*$. In any case the observations in all the regimes we discuss here are almost independent of M_* .

and accretion induced collapse (AIC) to a neutron star (e.g., Hillebrandt, Nomoto & Wolff 1984; Fryer *et al.* 1999; Dessart *et al.* 2006). All these explosions are expected to produce a rather constant ejecta velocity $(2E/M_{ej})^{1/2} \sim 10,000$ km/s, with explosion energy that ranges between 10^{51} erg in type Ia SNe, where the whole star is ejected ($M_{ej} \approx 1.4M_{\odot}$), to $10^{49} - 10^{50}$ erg in the other explosions, where only 1 – 10% of the stellar mass is ejected. There are also various observed SNe, which are probably generated by a range of white dwarf explosions. Some possible examples are SN 2005E, where $M_{ej} \approx 0.3M_{\odot}$ and $E \approx 4 \cdot 10^{50}$ erg (Perets *et al.* 2010), and SN2010X where $M_{ej} \approx 0.16M_{\odot}$ and $E \approx 1.7 \cdot 10^{50}$ erg (Kasliwal *et al.* 2010).

Since E/M_{ej} is similar for all these explosions, and the dependence of $\gamma_0\beta_0$ on E (or M_{ej}) when this ratio is constant, is very weak ($\propto E^{0.17}$), shock breakouts from all these types of white dwarf explosions produce a similar signature. The breakout Lorentz factor is $\gamma_0\beta_0 \approx 1 - 3$ for a white dwarf radius $3 - 10 \times 10^8$ cm. The total energy released in the breakout is $10^{40} - 10^{42}$ erg and the typical photon energy is \sim MeV. The breakout pulse is spread over $\sim 1 - 30$ ms having a peak luminosity of $\sim 10^{44}$ erg/s. These results for the breakout emission are different, especially in the temperature prediction, than those of Piro, Chang & Weinberg (2010) that found a type Ia breakout signal that peaks in the X-rays. The reason is that they assume thermal equilibrium behind the shock, which is not a good assumption for type Ia SNe (Nakar & Sari 2010), and that they ignore relativistic effects.

Note that this signature is expected only if the line of sight to the observer is transparent. In case that the explosion is triggered by white dwarf mergers, the optical depth of the debris that surrounds the exploding core can be high and the shock breakout will take place at much larger radius than the white dwarf radius, carrying out much more energy at a much lower velocity. For example, Fryer *et al.* (2010) find that a type Ia SN shock breakout in case of a double degenerate merger take place at a radius of $\sim 10^{13} - 10^{14}$ cm where the shock is Newtonian. Thus, a breakout emission from type Ia SN can potentially differentiate between single and double degenerate progenitor systems.

A flare of 10^{41} erg in MeV γ -rays can be detected easily if it takes place in the Milky way and possibly also in the Magellanic clouds. Such detection will provide a constrain on the on the exact explosion timing as well as on the radius of the exploding white dwarf and explosion energy. It may also shed light on the explosion mechanism. This detection channel is extinction free and may be the main detection channel in case that the burst take place in an obscured Galactic environment. Given the estimated Milky way rate of type Ia SNe (~ 0.01 yr $^{-1}$), the chance to detect such a burst in the coming decade is low, but not negligible. The rate of events such as SN 2010X is very hard to constrain observationally,

based on current optical surveys. In addition, there are no observational constraints on the rate of events with energy lower than 10^{50} erg, since their optical signature, where these events are searched for, is too weak and evolve too fast for detection. Similarly there are no tight theoretical constraints on the rates of .Ia SNe and AICs. Since the gamma-ray shock breakout signatures of all these events are similar and detectable for any galactic event, it is worth looking for them in the current data of satellites like BATSE, Konus-Wind, Swift and Fermi. In fact, Cline *et al.* (2005) find that the sub-sample of several dozen BATSE and Konus-wind GRBs, those with duration shorter than 0.1 s, shows a significant anisotropy with concentration of events in the galactic plane (but not towards the Galactic center). This sample of events may contain some already observed shock breakouts from white dwarfs.

5.2. Broad line Ibc SNe

Broad-line Ibc SNe are thought to be explosions of Wolf-Rayet progenitors, and in some cases are very energetic ($E \gg 10^{51}$ erg). Some Broad-line Ibc SNe are associated with GRBs (these are discussed later), but also those that are not associated with GRBs may produce a detectable γ -ray signal. For example SN 2002ap has a total explosion energy of $\sim 4 \cdot 10^{51}$ erg and an ejected mass of $\sim 2.5M_{\odot}$ (Maurer & Mazzali 2010), implying a mildly relativistic breakout with an energy output of $\sim 5 \times 10^{44} R_5^{0.7}$ erg within $\sim 10R_5$ s. At a distance of about 7 Mpc the observed fluence on earth is $\sim 10^{-7} R_5^{0.7}$ erg/cm². This fluence is detectable by sensitive gamma-ray detectors even for $R_* = R_{\odot}$ (although no large field-of-view detector with high sensitivity was operational in 2002). Note however, that if a dense wind surrounds the progenitor then the flare properties may be very different. The total emitted energy is higher while the typical photon energy may fall either within or below the observing band of soft γ -ray detectors.

5.3. Extremely Energetic SNe

Several types of extremely energetic SNe were detected in recent years. SN 2007bi has shown a typical velocity of 12,000 km/s and $> 50M_{\odot}$ of ejecta implying $E \gtrsim 10^{53}$ erg (Gal-Yam *et al.* 2009). Most of the optical luminosity observed in this SN is generated by radioactive decay. The radius of the progenitor is unknown, but since there is no trace of either hydrogen or helium its radius is most likely in the range, $R_* \sim 10^{11} - 10^{12}$ cm. These values implies $\gamma_0\beta_0 \approx 1$. Therefore the breakout is expected to produce a ~ 100 keV flash that lasts seconds to tens of seconds and carry an energy of $10^{44} - 10^{46}$ erg.

Quimby *et al.* (2009) reports on another type of extremely energetic SNe. They find a typical velocity of 14,000 km/s and $> 5M_{\odot}$ of ejecta implying $E \gtrsim 10^{52}$ erg. The observed optical energy in these explosions is not generated by radioactive decay and the possible sources are interaction of the ejecta with matter that moves more slowly or a central engine. In the former case, all the energy is given to the ejecta during the explosion and a bright shock breakout is expected. These explosions also do not show any trace of hydrogen and Helium implying that their progenitor is also rather compact. Assuming again $R_* \sim 10^{11} - 10^{12}$ cm the signature of the shock breakout from these events is similar to that expected from SN 2007bi.

An energy output of 10^{44} - 10^{46} in soft gamma-rays within a minute is detectable out to a distance of 3-30 Mpc. Therefore the detection of breakouts from such events is likely only if their rate is larger than $10^4 \text{ Gpc}^{-3} \text{ yr}^{-1}$. The rate of extremely energetic SNe is unknown, but it is most likely much lower than that since otherwise they should have been detected in optical searches in large bunnbers. We therefore do not expect current gamma-ray detectors to detect any of these SN types, although Nature may surprise us.

5.4. Low luminosity GRBs

The engine of long GRBs is thought to be produced during the collapse of a massive stellar core and thus to be launching relativistic jets into a stellar envelope. Bromberg, Nakar & Piran (2011) show that (at least in some) low-luminosity GRBs it is highly unlikely that the jets successfully punch through the star and emerge in order to produce the observed γ -ray emission. In that case what can be the source of the observed gamma-rays?

There are several common features observed in all low luminosity GRBs. All show $10^{48} - 10^{50}$ erg of gamma-rays or hard X-rays that are emitted in single pulsed light curves showing a spectra that become softer with time (see Kaneko *et al.* 2007 and references therein). They also show radio afterglows that indicate on similar energy in mildly relativistic ejecta (Kulkarni *et al.* 1998; Soderberg *et al.* 2004, 2006). This energy is only a small fraction of the total energy observed in the associated SNe ($10^{52} - 10^{53}$ erg). The smooth light curves and the energy coupled to a mildly relativistic ejecta motivated several authors to suggest that the origin of this emission is shock breakouts (Kulkarni *et al.* 1998; Tan, Matzner & McKee 2001; Campana *et al.* 2006; Waxman, Mészáros & Campana 2007; Wang *et al.* 2007). Later, Katz, Budnik & Waxman (2010) have shown that the deviation from thermal equilibrium can explain the observed γ -rays, compared to the expected X-rays when equilibrium is wrongly assumed. However, the question whether shock breakouts are indeed the source of low-luminosity GRBs remained open, mostly since there was no quan-

titative model that could answer whether shock breakouts can indeed explain even the most basic observables, such as energy, temperature and time scales, of these bursts. Below, we use our model to answer this question. In section 4 we have shown that this three observed scales, E_{bo} , T_{bo} and t_{bo}^{obs} , over-constrain the physical parameters since they depend in the case of a quasi-spherical relativistic breakout only on two parameters, $\gamma_{f,0}$ and the breakout radius. Thus, in addition to finding the value of these two physical parameters, the three observables must satisfy the relativistic breakout relation (equation 18). This can be used as a test that any flare that originates from quasi-spherical relativistic breakout must pass. This result is generic as it is independent of the pre-breakout density profile (as long as it is steeply decreasing). Below we apply this test to the four observed low-luminosity GRBs

The four well studied low luminosity GRBs are divided into two pairs with very different properties. GRBs 060218 and 100316D are soft and very long. Their rather soft spectrum is best fitted at early time with a cut-off around a peak energy of ~ 40 keV (Kaneko *et al.* 2007; Starling *et al.* 2011) and both emitted a total energy of $\sim 5 \times 10^{49}$ erg. Plugging these values into equation 18 we find that if these bursts are flares from quasi-spherical breakouts then they should have durations of ~ 1500 s, which is indeed comparable to the observed durations of these two bursts. The breakout radius in this model is $\sim 5 \times 10^{13}$ cm and $\gamma_{f,0} \approx 1$ implying $\gamma_0 \beta_0 \approx 0.3 - 1$. In the case of GRB 060218, radio emission suggest breakout velocities on the upper side of this range (Soderberg *et al.* 2006). The large radius inferred by the long duration implies that the breakout is most likely not from the edge of the progenitor star, but from an opaque material thrown by the progenitor to large radii (possibly a wind) prior to the explosion (Campana *et al.* 2006). The fact that a spherical model provides a good explanation implies that the suggested a-sphericity (Waxman, Mészáros & Campana 2007), if exist, does not play a major role in determining the burst duration. Note though, that it may play an important role in shaping the breakout spectrum above and below T_{bo} and the entire light curve following the breakout flare.

The other two low-luminosity GRBs, 980425 and 031203, are relatively hard, $T \gtrsim 150$ keV, and not particularly long, ≈ 30 s. Their hard spectrum made it difficult to explain these events as shock breakouts, before it was realized that the radiation will be away from thermal equilibrium⁷. Plugging 10^{48} erg and 150 keV, the total energy and typical frequency

⁷For example, Tan, Matzner & McKee (2001), which consider shock breakout as the source of energy of GRB 980425, add an interaction of the expanding gas with external mass shell, at a radius where the gas is optically thin, in order to reprocess the gas energy into internal energy and radiate it non-thermally. Wang *et al.* (2007) suggested that the gamma-rays are produced by bulk comptonization of low energy thermal photons during the breakout. A process that does not work at the relevant temperatures due to Compton loses (Katz, Budnik & Waxman 2010)

of GRB 980425, into equation 18 results in a breakout duration of ~ 10 s, comparable to the observed one. The breakout radius in that case is $\sim 6 \times 10^{12}$ cm and the final Lorentz factor is⁸ $\gamma_{f,0} \approx 3$. The associated supernova of GRB 980425, SN 1998bw, was very energetic exhibiting $M_{ej} \approx 15M_{\odot}$ and a kinetic energy of $\approx 0.5 \times 10^{53}$ erg (Iwamoto *et al.* 1998). Interestingly, Setting $R_* \sim 6 \times 10^{12}$ and $M_{ej} = 15M_{\odot}$ in equation 26 we find that an explosion energy of $E \approx 3 \times 10^{53}$ produces a breakout with the observed properties of GRB 980425. This energy is slightly larger than the one seen in the SN 1998bw, but it is close enough to suggest that the energy source of the SN explosion is also the one of the shock breakout. The mild discrepancy between the two energies may be a result of the actual stellar density profile (compared to the power-law with $n = 3$) or of a deviation of the explosion from sphericity. Using the energy and typical frequency of GRB 031203, 5×10^{49} erg and > 200 keV, equation 18 predicts a duration $\lesssim 35$ s. The observed duration, ~ 30 s is consistent with this value and indicates that the actual temperature is not much higher than 200 keV. The breakout radius in that case is $\sim 2 \times 10^{13}$ cm and the final Lorentz factor is $\gamma_{f,0} \approx 5$. The SN associated with GRB 031203, SN 2003lw, is similar in ejected mass and slightly more energetic than SN 1998bw (Mazzali *et al.* 2006). Setting $R_* \sim 2 \times 10^{13}$ and $M_{ej} = 15M_{\odot}$ in equation 26 we find that an explosion energy of $E \approx 10^{54}$ produces a breakout with the observed properties of GRB 031203. Again this value is larger by about an order of magnitude compared to the one observed in the associated SN.

The breakout radii that we find for GRBs 980425 and 031203 are larger than those of typical Wolf-Rayet stars, the probable progenitors of GRB-SNe. This implies that either the breakout is not from the edge of the progenitor star (e.g., from a wind) or that low-luminosity GRB progenitors are larger than commonly thought (which may help chocking the relativistic jet). Finally, a prediction of relativistic shock breakouts is a significant X-ray emission, that can be comparable in energy to that of the breakout γ -ray flare. The X-rays are emitted during the spherical phase, over time scale that is significantly longer than that of the breakout emission. GRB 031203 has shown a bright signal of dust scattered X-rays, which indicates on ~ 2 keV emission with energy comparable to that of the gamma-rays, during the first 1000 s after the burst (Vaughan *et al.* 2004; Feng & Fox 2010). These values fit the predictions of a breakout model. For example, if the breakout is from a stellar surface at $R_* = 2 \times 10^{13}$ and $M_{ej} = 15M_{\odot}$, then equations 30-32 predict a release of $\sim 10^{50}$ erg at ~ 2 keV within the first hour after the burst.

⁸ Although this value of the Lorentz factor is below the lower-limit derived by Lithwick & Sari (2001) for GRB 980425, there is no contradiction between the two studies. The reason is that Lithwick & Sari (2001) derived their limits assuming that there is a hard spectral power-law that extends to very high energies, similarly to the one observed in long GRBs. However such power-law does not exist in the case of relativistic breakout and therefore their constraints are not applicable to the model that we discuss here.

To conclude, we find that the energy, temperature, and time scales of all four low-luminosity GRBs are explained very well by shock breakout emission and that all of them satisfy the relativistic breakout relation between E_{bo} , T_{bo} and t_{bo}^{obs} (equation 18). These observables are largely independent of the pre-shocked density profile and are therefore applicable also for a breakout from a shell of mass ejected by the progenitor prior to the explosion and possibly to a breakout from a wind. Breakout emission can also explain many other observed properties, such as the smooth light curve, the afterglow radio emission, the small fraction of the explosion energy carried by the prompt high energy emission, and the delayed bright X-ray emission seen in GRB 031203. We therefore find it to be very likely that indeed all low-luminosity GRBs are relativistic shock breakouts.

5.5. GRB 101225A

GRB 101225A is a peculiar explosion showing an \sim hour long smooth burst of gamma-rays and X-rays followed by a peculiar IR-UV afterglow, which is consistent with a blackbody emission. Its origin is unclear. Thöne *et al.* (2011) argue for a cosmological origin and estimate its redshift to be ≈ 0.3 , while Levan & Tanvir (2011) argue that its origin is local. Here we do not attempt to determine which, if any, of the two views is correct. Instead, given the similarity of the high energy emission of GRB 101225A to some low-luminosity GRBs, we ask whether shock breakout can be the source of this emission, assuming that the burst is cosmological and that the redshift estimate of Thöne *et al.* (2011) is correct.

We test if its observed properties satisfy equation 18, which with a total energy of $\sim 10^{51}$ erg and gamma-ray spectrum that peaks at ~ 40 keV predicts a duration of $\sim 10^4$ s. This value is consistent with the constraints on the actual duration of the flare (Thöne *et al.* 2011). The resulting breakout radius is $\sim 3 \times 10^{14}$ cm. Interestingly Thöne *et al.* (2011) present a model that attempts to explain the IR-UV afterglow by an expanding opaque shell with velocities at least as high as $\beta \approx 1/3$ and initial radius of 2×10^{14} cm. A breakout of a shock that accelerates such a shell produces a high energy signal that is similar to the observed one. We therefore conclude that if Thöne *et al.* (2011) redshift estimate is correct, then a relativistic shock breakout probably provides the simplest explanation for the high energy emission.

5.6. Long gamma-ray bursts

In the colapsar model of long GRBs the relativistic jets pierce through the envelope and emerge in order to produce the observed gamma-ray emission. During their propagation the jets drive mildly relativistic bow shocks into the envelope and inflate a cocoon that collimates the jets to an even narrower angle than their initial launching opening angle (e.g., MacFadyen & Woosley 1999; Matzner 2003; Morsony, Lazzati & Begelman 2007; Mizuta & Aloy 2009). Recently, Bromberg *et al.* (2011) derived an analytic description of this interaction while the jet is propagating deep in the envelope, before it encounters the sharp density gradient near the stellar edge.

The term shock breakouts from GRBs is used in the literature to describe several different phenomena. Here, similarly to the rest of the paper, we focus on the emission that is escaping from the breakout layer after the forward shock, which is driven by the jet head and the cocoon into the stellar envelope, breaks out. The structure of the shock that is driven into the stellar envelope is highly non spherical but at the time of the size of causally connected regions is small an a local spherical approximation is appropriate. Thus, the theory discussed above, together with the model presented in Bromberg *et al.* (2011) and the results seen in numerical simulations (e.g., Mizuta & Aloy 2009), can be used to obtain a rough idea of the expected breakout signal.

We consider a typical GRB jet with a half-opening angle $\theta_0 = 0.1$ rad and a total luminosity of 10^{50} erg/s. The jet is launched continuously into a $5M_\odot$ progenitor with a radius of $5R_\odot$. At the point that the jet reaches the steep density drop (i.e. $r \sim R_*/2$) its velocity approaches the speed of light and it is narrowly collimated to an angle of $\sim \theta_0^2$ (Bromberg *et al.* 2011). The opening angle of the cocoon at this point is $\approx \theta_0$. From that point the jet head and the cocoon shock starts accelerating towards the stellar edge and to expand sideways. Numerical simulations show that the head is expanding sideways, becoming comparable in size to the cocoon which does not spread significantly (e.g., figure 17 in Mizuta & Aloy 2009). Once the fractional distance of the shock to the edge, z/R_* , is comparable to the shock opening angle, in our case $\sim \theta_0 = 0.1$, then the shock lose causal connection with the sides and it can be approximated by the spherical solution of an accelerating blast wave (Johnson & McKee 1971). Taking $\gamma_s \beta_s(z = 0.1R_*) \approx 1$ and using the relation $\gamma_s \beta_s \propto z^{-0.69}$ with equation 5, we obtain $\gamma_0 \approx 10$. This value is larger than $\gamma_{0,s}$ so the breakout shell accelerates also after the transition to the spherical phase. We do not provide here the solution to such case, but the initial energy in the breakout shell is $\sim 10^{48}(R_*/5R_\odot)^2$ erg and its energy increases during acceleration by at most an order of magnitude. Assuming that there is no significant wind surrounding the progenitor, this energy is released by a short, \sim ms, pulse of $>$ MeV photons.

The main distinguishable feature of the breakout pulse is its harder spectrum and low fluence compared to the total burst emission. The pulse luminosity depends strongly on the progenitor size. If the progenitor is rather large, $R_* \gtrsim 5R_\odot$, then the breakout can be detected by Swift and Fermi up to a redshift ~ 0.1 . If it is much smaller, then the breakout is too faint for detection. Even in cases that the breakout is detectable, it may be difficult to separate it from the GRB itself. Nevertheless, it is worth looking for an initial hard spike in nearby GRBs. Interestingly, there is a sub-sample of BATSE GRBs that show an initial short hard spikes followed by a softer burst (Norris & Bonnell 2006).

6. Summary

We calculate the emission from mildly and ultra relativistic spherical shock breakouts. First, we determine the pre and post breakout hydrodynamical evolution. This requires knowing the shock width, which we obtain by deriving an improved analytic description to the structure of the transition layer in relativistic radiation mediated shocks, based on the results of Budnik *et al.* (2010). After obtaining the hydrodynamical evolution we follow the temperature and opacity evolution to find the observed light curve. Our calculations are applicable to stellar explosions with shock velocity larger than $0.5c$, in which the breakout shell ends its acceleration during the planar phase, i.e., its final Lorentz factor $\lesssim 30$ for typical stars. We consider only cases where the progenitor wind has no effect on the observed emission.

A relativistic radiation mediated shock brings the gas to a roughly constant rest frame temperature, ~ 200 keV, and loads it with pairs (Katz, Budnik & Waxman 2010; Budnik *et al.* 2010). Photons remain confined to the post-shock expanding gas as long as pairs keep it optically thick. A significant number of photons are released towards the observer only from optically thin regions, which has a gas rest frame temperature $\lesssim 50$ keV (low pair load) and unloaded pair gas opacity, τ , smaller than unity. Our solution of the shock structure shows that the shock width is significantly smaller than $\tau = 1$, implying that the shock accelerates also shells with $\tau \leq 1$, which become transparent once their pairs annihilate. Among these shells the one that carries most of the energy is the most massive one, i.e., $\tau \approx 1$, which we refer to as the breakout shell. In the regime that we consider the breakout shell becomes transparent during the planar phase (i.e., before its radius doubles) and after it achieves its final Lorentz factor. Light travel time and relativistic effects dictate that the emission of the breakout shell, once it become transparent, dominates during the entire planar phase. Slower shells release their photons only during the spherical phase. The processes described above produce the main following observables:

(i) *A γ -ray flare with a typical photon energy that ranges between ~ 50 keV and ~ 2 MeV. The flare typically contains a small fraction of the explosion energy and it may be significantly shorter than the progenitor light crossing time.* This flare is generated by the breakout shell when it becomes transparent.

The Energy, temperature and duration of the flare depend only on two parameters, $\gamma_{f,0}$ and the breakout radius. Thus, in addition to providing the value of these two physical parameters, the three observables must satisfy the relativistic breakout relation (equation 18). This relation can be used to test if an observed flare may have been produced by a quasi-spherical relativistic breakout. Note, that our calculation of the breakout flare energy, temperature and duration are independent of the exact profile of the pre-breakout density, as long as it is steeply decreasing.

(ii) Due to light travel time effects, the breakout flare contains photons from faster and lighter shells, producing a high energy power-law spectrum, $\nu F_\nu \propto \nu^{-0.74}$. It also contains photons emitted by the breakout shell after it becomes transparent and cools adiabatically, producing a low-energy power-law spectrum, $\nu F_\nu \propto \nu$. Both power-laws are limited to about one order of magnitude in frequency.

(iii) The flare ends with a sharp decay at a time that coincides with the transition to the spherical phase. If the breakout is ultra relativistic (i.e., $\gamma_{f,0} \gg 1$) then $L \propto t_{obs}^{-2}$ and $T \propto t_{obs}^{-1}$, until the spherical phase emission dominates.

(iv) After the flare ends, spherical evolution dictates a steady decay of the luminosity. At first, while relativistic shells dominate the emission, $L \propto t_{obs}^{-1.1}$. Later, at t_{NW}^{obs} , once Newtonian shells become transparent $L \propto t_{obs}^{-0.35}$. Note that during the latter phase the total emitted energy increases with time.

(v) The post flare temperature decays at first at a steady rate of about $t_{obs}^{-0.6}$, generated during the spherical phase. A sharp drop in the temperature is observed when shells which were not loaded by pairs during the shock crossing ($v_s \lesssim 0.5$) dominate the emission. The drop is typically from the X-ray range to the UV, and it is observed at about $10 t_{NW}^{obs}$.

(vi) The energy emitted before the steep temperature drop is often comparable to that emitted during the breakout flare. Thus, *a signature of relativistic breakout in many scenarios is a bright and short γ -ray flare and a delayed X-ray emission with comparable energy.*

We apply our model to a range of observed and hypothesized explosions finding the following predictions:

- **Type Ia SNe and other white dwarf explosions:** A range of white dwarf explosions, including observed Ia SNe and SNe 2005E & 2010X as well as hypothesized .Ia SNe and AIC, are all predicted to produce a similar breakout emission. A $\sim 1 - 30$ ms pulse of $10^{40} - 10^{42}$ erg composed of \sim MeV photons. Such pulse from a Galactic explosion is detectable, implying a low, yet not negligible, chance to detect such flare

from type Ia SNe in the coming decade. The rate of the other types of explosions are not known. Moreover there is almost no correlation between the properties of the γ -ray flare and the luminosity of the optical emission. Implying that Galactic breakouts from white dwarf explosions may have already been observed, but not recognized as such. In this context the suggested sub-class of very short GRBs (Cline *et al.* 2005) may contain such events.

- **Broad-line Ib/c SNe:** Their progenitor is compact enough and the energy of some is high enough to form mildly relativistic breakouts. For example SN2002ap-like events are predicted to produce a detectable breakout γ -ray signal in case that there is no thick wind surrounding the progenitor.
- **Extremely energetic SNe:** SNe such as 2007bi (Gal-Yam *et al.* 2009) and explosions of the type reported by Quimby *et al.* (2009), are energetic enough to generate mildly relativistic breakouts in very massive stars. These breakouts are likely to release $10^{44} - 10^{46}$ erg in soft γ -rays over seconds to minutes. Such flashes are detectable in the nearby Universe, however these are rare events, and the chance to have one in a detectable distance is low.
- **Low luminosity GRBs:** The origin of low-luminosity GRBs is unknown. Bromberg, Nakar & Piran (2011) show that low-luminosity GRBs are very unlikely to be produced by relativistic jets that pierce through their host star, which is a necessary ingredient of the collapsar model for long GRBs. In that case what can be the source of the high energy emission? The smooth light curve and the mildly relativistic ejecta that generate the radio emission of GRB 980425 lead Kulkarni *et al.* (1998) and Tan, Matzner & McKee (2001) to suggest that relativistic shock breakouts are related to low luminosity GRBs. The detection of a thermal component in GRB 060218 (Campana *et al.* 2006) and of additional low luminosity GRBs with similar properties to GRB 980425, supported this suggestion (Waxman, Mészáros & Campana 2007; Wang *et al.* 2007), especially after it was realized that the shock emission strongly deviates from thermal equilibrium (Katz, Budnik & Waxman 2010). However, there was no quantitative model of the breakout emission that includes deviation from thermal equilibrium and gas-radiation relativistic dynamics, which could test this suggestion. Moreover, it was unclear whether shock breakouts can generate the energies seen in these bursts (especially GRBs 980425 and 031203 due to their shorter duration).

We find that relativistic shock breakouts can explain very well the energy, temperature and time scales of the prompt emission of all observed low-luminosity GRBs. Moreover, all four known bursts satisfy the relativistic breakout relation between these three observables (equation 18). We also show that relativistic breakouts provide a natural

explanation to the low gamma-ray luminosity compared to the total explosion energy and to the hard to soft spectral evolution (including the late energetic X-ray emission that inferred from dust echoes of GRB 031203). We therefore find shock breakouts to be the most promising sources of all low-luminosity GRBs. The connection between low-luminosity and long GRBs (both share a relation to a similar type of SNe), suggests that the energy source for the shock breakout are choked relativistic jets that are launched by a GRB engine but fail to punch through the star.

In this context we examine the high energy emission of GRB 101225A, whose origin (local or cosmological) is undetermined yet, but its prompt emission resemble low-luminosity GRBs. We find that if the redshift determination of Thöne *et al.* (2011), $z \approx 0.3$, is correct then this burst also successfully passes the test set by equation 18, suggesting that the source of its high energy emission is a mildly relativistic shock breakout at a relatively large radius of $\sim 3 \times 10^{14}$ cm.

- **Long GRBs:** According to the popular view of long GRBs, relativistic jets drill their way through the progenitor envelope driving a forward shock into it. We estimate that a typical value of the initial breakout Lorentz factor of this shock is $\gamma_0 \sim 10$. Our theory cannot determine its final Lorentz factor, since it ends its acceleration after the evolution enters the spherical regime. Nevertheless, this value of γ_0 indicates that shock breakout from long GRBs release $\sim 10^{48} (R_*/5R_\odot)^2$ erg in a short pulse of $> \text{MeV}$ photon. This pulse is too dim to be observed in the typical redshift of long GRBs, but it may be observable in nearby $z \sim 0.1$ GRBs.

We conclude that relativistic shock breakouts are a generic process for the production of gamma-ray flares, which opens a new window for the study and detection of a variety of stellar explosions.

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APPENDIX - The transition layer structure of relativistic radiation mediated shocks

Here we derive an analytic description of the transition layer of relativistic radiation mediated shocks, based on the results of Budnik *et al.* (2010). We use mostly similar notations to those of Budnik *et al.* (2010) and carry out the calculation in the shock frame, where the flow is assumed to be in a steady state. We approximate the gas-radiation in

the transition region as two counter flows. One is a flow of photons with energy $\sim m_e c^2$ that are generated in the immediate downstream and flow towards the upstream at a drift velocity $\sim c$. The other is a gas flow that is moving towards the downstream. This is a flow of the fluid that is shocked, which is moving with a Lorentz factor Γ_u in the far upstream (this is the shock Lorentz factor) and is slowed down in the transition region to $\beta = 1/3$ (i.e., $\Gamma \approx 1$). The downstream moving fluid is composed of protons with a proper density n and pairs and photons with a proper density $n_{\pm,\gamma}$. The photons in this flow were part of the photon flow that moves towards the upstream, before they were scattered once by the fluid leptons and joined the fluid motion towards the downstream⁹. We are interested in the shape of the transition, i.e., Γ and the pair fraction as a function of optical depth and physical length. Our final goal is to find the shock width in the upstream frame in units of the pre-shock (i.e., with no pairs) upstream Thomson optical depth.

The shock is generated by the interaction of the two counter-streaming flows. Every collision between upstream moving photon and a downstream moving fluid lepton[photon] transfers energy and momentum to the photon[created lepton] and makes it joining the downstream moving fluid. The temperature of the pairs and fluid photons is $\sim \Gamma m_e c^2$, as seen in the fluid rest frame. In a steady state the flux of photons that are moving towards the upstream, $n_{\gamma \rightarrow us}$, is equal to the number flux of pairs and photons that are moving with the fluid (assuming that the later dominate over the proton number flux), i.e., $n_{\gamma \rightarrow us} \approx \Gamma \beta n_{\pm,\gamma}$. Now, since upstream moving photon that is scattered is joining the downstream moving fluid, $d(\Gamma n_{\pm,\gamma}) = -n_{\gamma \rightarrow us} d\tau_s$, where τ_s is the optical depth for photons moving toward the upstream, defined to be increasing in the upstream direction¹⁰. Defining $x_{\pm,\gamma} \equiv n_{\pm,\gamma}/n$ and assuming $\Gamma^2 \gg 1$ (i.e., $\beta \approx 1$), we obtain:

$$\frac{dx_{\pm,\gamma}}{d\tau_s} = -x_{\pm,\gamma}. \quad (\text{A-1})$$

Number and momentum flux conservation imply:

$$n_u \Gamma_u \beta_u = n \Gamma \beta \quad (\text{A-2})$$

$$\Gamma_u^2 \beta_u n_u m_p c^2 = \Gamma^2 \beta n c^2 (m_p + 4x_{\pm,\gamma} \Gamma m_e) \quad (\text{A-3})$$

In the momentum equation we assume that the momentum flux of the photon field that streams towards the upstream is negligible, which is true as long as $\Gamma^2 \gg 1$. We also assume

⁹Protons and pairs are assumed to be coupled by collective plasma processes on scales much shorter than any scale on interest in the system and thereby can be treated as a single fluid. The photons in this flow are having the same downstream drift motion, but are not directly coupled to the protons and leptons.

¹⁰Our definition of τ_s is similar to $-\tau$ in the notation of Budnik *et al.* (2010)

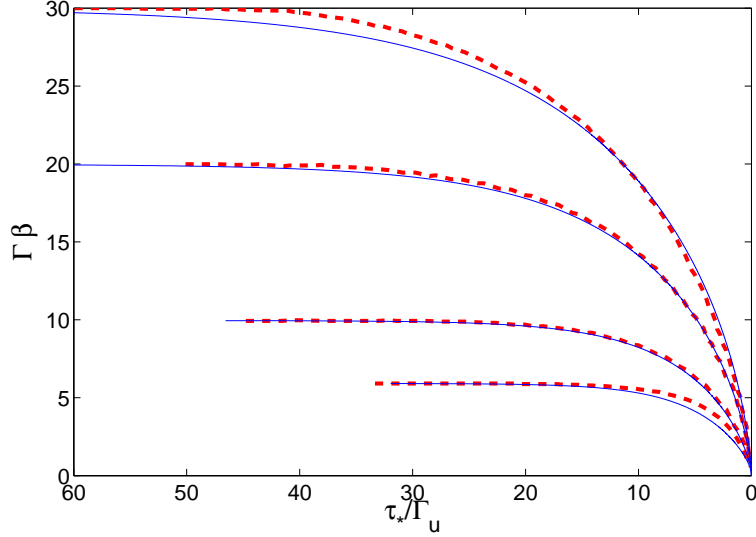


Fig. 2.— Comparison of the shock structure for four values of Γ_u (6, 10, 20 & 30), obtained by integrating over equation A-6 (using $\tau_* = \tau_s \frac{\sigma_T}{\sigma_{KN}(1.4\Gamma^2)}$), [*thin solid line*], to the numerical results of Budnik *et al.* (2010) (their figure 6), [*thick dashed line*]

$x_{\pm,\gamma} \gg 1$. These equations imply:

$$x_{\pm,\gamma} = \frac{m_p}{4m_e} \frac{\Gamma_u - \Gamma}{\Gamma^2} \quad (\text{A-4})$$

Therefore:

$$\frac{d\Gamma}{d\tau_s} = \frac{d\Gamma}{dx_{\pm,\gamma}} \frac{dx_{\pm,\gamma}}{d\tau_s} = \Gamma \frac{\Gamma_u - \Gamma}{2\Gamma_u - \Gamma} \quad (\text{A-5})$$

If we define τ_* as the Thompson optical depth of a photon moving toward the upstream then $\tau_* \approx \tau_s \frac{\sigma_T}{\sigma_{KN}(\Gamma^2)}$, where σ_T is the Thomson cross-section and $\sigma_{KN}(\Gamma^2)$ is the Klein-Nishina cross section of electron at rest and photon with energy $\Gamma^2 m_e c^2$. Therefore

$$d\tau_* = \frac{\sigma_T}{\Gamma \sigma_{KN}(\Gamma^2)} \frac{2\Gamma_u - \Gamma}{\Gamma_u - \Gamma} d\Gamma \quad (\text{A-6})$$

Integrating this equation starting at the end of the transition layer, i.e., $\Gamma(\tau_s = 0) = 1$ results in the structure of the transition layer. Comparison of this structure to the numerical results of Budnik *et al.* (2010) is presented in figure 2.

In order to find the width of the shock in the upstream frame we use the relation

$$d\tau_* = \Gamma_u n_u x_{\pm,\gamma} \sigma_T dz' \quad (\text{A-7})$$

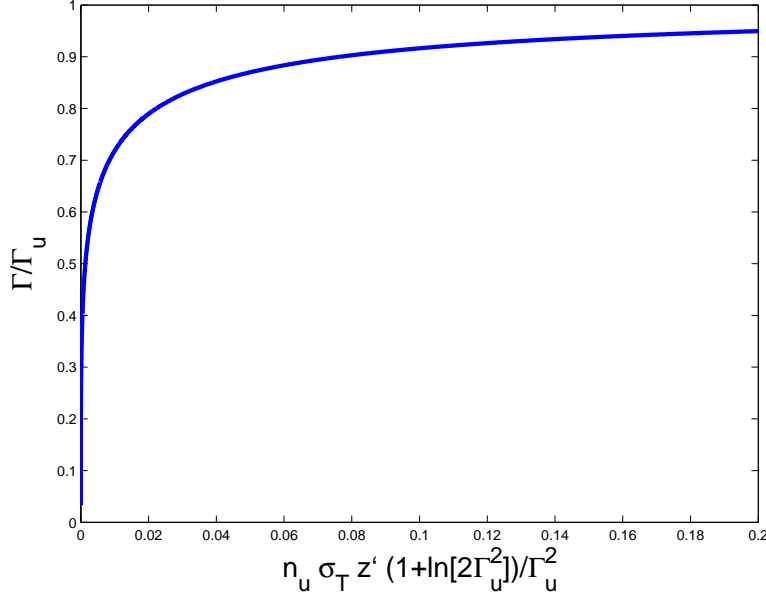


Fig. 3.— The structure of the shock transition region of relativistic radiation mediated shocks, presented in an optical depth scale $(n_u \sigma_T z' (1 + 2 \ln[\Gamma_u^2])/\Gamma_u^2)$, which makes it universal. i.e., the value of Γ/Γ_u is independent of Γ_u (for values larger than $1/\Gamma_u$).

where z' is the length coordinate (positive towards the upstream) in the shock frame. Therefore

$$n_u \sigma_T dz' = \frac{4m_e}{m_p} \frac{2\Gamma_u - \Gamma}{(\Gamma_u - \Gamma)^2 \Gamma_u} \frac{\sigma_T}{\sigma_{KN}(\Gamma^2)} \Gamma d\Gamma \quad (\text{A-8})$$

Integrating $d\Gamma$ from the shock towards the upstream, the value of Γ/Γ_u (for values larger than $1/\Gamma_u$) has a universal profile (independent of Γ_u) as a function of $n_u \sigma_T z' (1 + 2 \ln[\Gamma_u^2])/\Gamma_u^2$. This profile is presented in figure 3. This profile implies that the shock width in unit of pair unloaded Thomson optical depth in the shock frame is roughly¹¹, $\propto \Gamma_u^2$. The value of the proportionality coefficient is somewhat arbitrary and it depends on the value of Γ/Γ_u that defines the “shock width”. We chose $\Gamma/\Gamma_u = 0.9$ obtaining $n_u \sigma_T z' \approx 0.01 \Gamma_u^2$ (the value of the logarithmic factor is 5 – 10 for $\Gamma_u = 2 - 10$). Choosing a different value of Γ/Γ_u , to define

¹¹Note that here we find the shock width in pre-shock, *without pairs*, Thomson optical depth, which can be directly related to physical width for a given upstream density. Budnik *et al.* (2010) find the shock width in units of total Thomson optical depth, *including pairs*, of a photon that crosses the transition layer towards the upstream, τ_* . As it turns out, in both cases the width $\propto \Gamma^2$ but the proportionality coefficient is different (Budnik *et al.* 2010 find $\Delta\tau_* \approx \Gamma^2$).

the shock width, changes the coefficient, which in turn has a very weak effect on equation 8. Now, since the length scale in the upstream frame, z , is shorter by a factor Γ_u than in the shock frame (i.e., $z' = \Gamma_u z$), and defining $\tau_u = n_u \sigma_T z$, we approximate the shock width in units of Thomson optical depth of the pre-shock upstream (i.e., with no pairs) as:

$$\Delta\tau_u \sim 0.01\Gamma_u \quad (\text{A-9})$$

Note that our analytic description is applicable to shocks where $\Gamma_u^2 \gg 1$ and are therefore not directly applicable to mildly relativistic shocks. We do expect however that pair creation in these shocks, which increases the optical depth per proton in the immediate downstream by a factor of the order of m_p/m_e , will result in $\Delta\tau_u \ll 1$ and that equation A-9 can be extrapolated to the mildly relativistic regime.

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